Perception and The Golden Section

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It is widely believed that humans perceive time exponentially rather than linearly. e.g. A periodic set of events spread over a large time period will appear to get further apart, the longer they are observed. Thus, through mapping a set of linear time-values onto an exponential curve one could closer approximate the human perception of time.

Throughout the centuries it has been thought that the Golden Section has proportional qualities ideal for architecture, art and music.

As a purely academic exercise I decided to combine the two above ideas into a single approach.

If humans perceive time exponentially, then the middle of a piece of music as measured by a clock (Clock Time) would be at a different point to where it was perceived (Perceptual Time).

If human perception is exponential, the halfway point in Perceptual Time will always appear later in the piece than in Clock Time. The Golden Section occurs 61.8% of the way into a piece - after the halfway mark.

SUPPOSITION: The Golden Section is successful in structuring music because it approximates the perceptual halfway mark. i.e. An audience perceives the Golden Section as the halfway point in a piece of music because of the exponential nature of their perception of time.

If an exponential curve could be created in which the halfway point was exactly the golden section, then this would create a synthesis of the two approaches and might create a system of proportioning music closer to human perception.

Let;
The beginning of a piece of music = 0
The end of a piece of music = 1

Then:
The middle of a piece of music = 0.5
The Golden Section of a piece of music = 0.618034…

I created an equation which produces an exponential curve and satisfies the conditions 0=0, 1=1 and 0.5=0.618034.

The equation for this curve is:

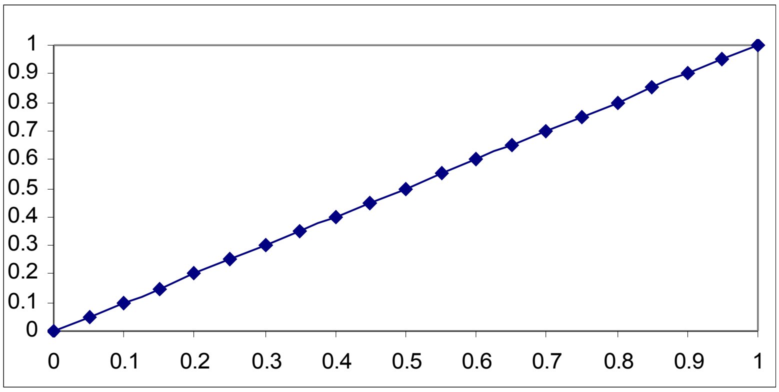
$$4\left(0.25-(n-0.5)^2\right)\times\left(\frac{1}{\left(\frac{1+\sqrt{5}}{2}\right)}-0.5\right)+n$$

where $0 \leqslant n \leqslant 1$

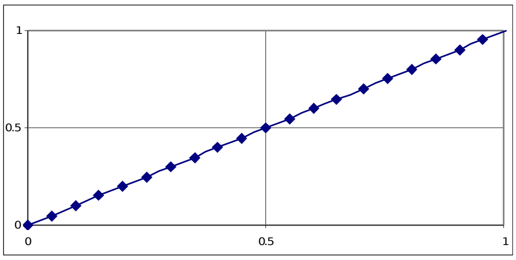
The differences between this equation, an alternative exponential equation ($y=x^2$) and a linear equation can be seen in the graphs below:

Fig 1. Showing linear, Golden Section and square root graphs(a), and their perceptual halfway point (b). All graphs plot Perceptual time (Y-axis) against Clock Time (X-axis).

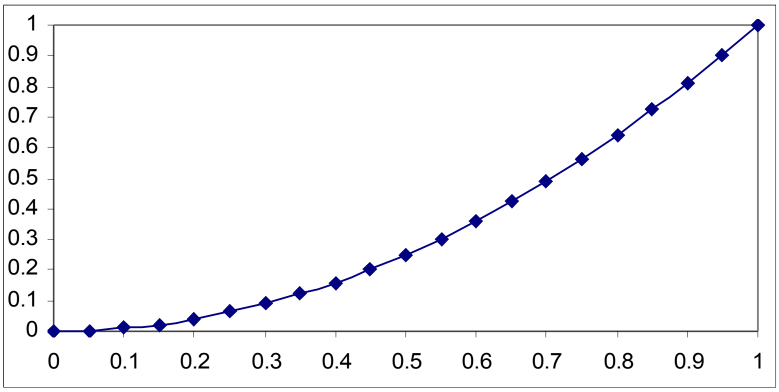
1(a) Linear equation: $x=y$



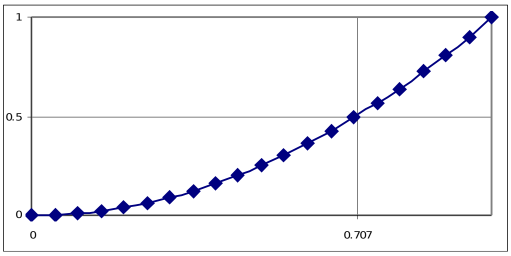
1(b) Perceptual midpoint = 0.5



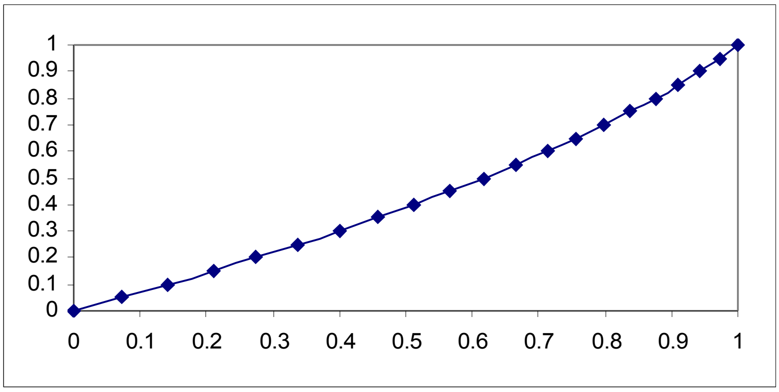
2(a) $y=x^2$



2(b) Perceptual midpoint=0.7071..



3(a) My Golden Section equation



3(b) Perceptual midpoint = 0.618..

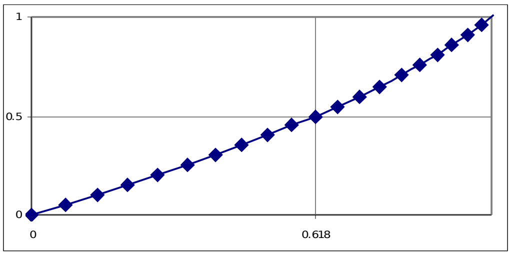


Fig 2. Numbers used to plot the graphs above

y=x	*y=x2*	Golden Section Equation
0	0	0
0.05	0.0025	0.072426458
0.1	0.01	0.142492236
0.15	0.0225	0.210197334
0.2	0.04	0.275541753
0.25	0.0625	0.338525492
0.3	0.09	0.399148551
0.35	0.1225	0.45741093
0.4	0.16	0.513312629
0.45	0.2025	0.566853649
0.5	0.25	0.618033989

y=x	*y=x2*	Golden Section Equation
0.55	0.3025	0.666853649
0.6	0.36	0.713312629
0.65	0.4225	0.75741093
0.7	0.49	0.799148551
0.75	0.5625	0.838525492
0.8	0.64	0.875541753
0.85	0.7225	0.910197334
0.9	0.81	0.942492236
0.95	0.9025	0.972426458
1	1	1

Uses

This equation can be used to dictate the proportions of a piece by either:

1. Using the formula to generate pre-compositional proportions based upon the Golden Section:
    i. Suppose one wanted a piece in 4 equal sections with sectional divisions at: ¼, ½, and ¾.
    b. These sectional divisions are converted into numbers between 0 and 1, i.e: 0.25, 0.5, and 0.75
    c. These numbers are then substituted into the formula, replacing *n*. This gives the following results:
        0.25=0.3262..
        0.5=0.618034.. (The Golden Section)
        0.75=0.83825..
    d. These numbers can then be multiplied by the length of the piece in order to calculate the time at which they will appear. If the piece will last 5 minutes (300 seconds) then:
        300 seconds x 0.326 = 97.8 seconds (1 minute 37.8 seconds)
        300 seconds x 0.618 = 185.4 seconds (3 minute 5.4 seconds)
        300 seconds x 0.838 = 251.6 seconds (4 minute 11.6 seconds)
    Now all sectional divisions have been shifted to more perceptually accurate positions (with the mid-point being at the Golden Section) meaning that, theoretically, all sections should sound approximately the same length.

2. Converting times of a piece into a number between 0 and 1 and using the equation to adjust the proportions to more perceptually accurate ones.
    a. A piece is 5 minutes long (300 seconds), with important events occuring at:
        1 minute 15 (75 seconds)
        2 minutes 30 (150 seconds) and
        4 minutes 15 (225 seconds).
    b. The above times (in seconds) should be divided by the overall time (in seconds) to get a number between 0 and 1 which can be used in the equation:
        75/300 = 0.25
        150/300 = 0.5
        225/300 = 0.75
    Instructions 1c and 1d should then be followed.

This is a technique which appeals to mathematical beauty rather than scientific rigour and is designed merely as an interesting way of structuring pieces through the combination of two different approaches.

I suspect its perceptual effects will be inconclusive at best and it is not proposed as a solution to the ever-pertinent questions related to how audiences temporally perceive music.

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